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STUDY OF LAMINAR FLOW IN A LARGE DIAMETER ANNULUS WITH TWISTED TAPE INSERTS

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ABSTRACT

In the present work an experimental study has been conducted to investigate the variation of the friction factor with Reynolds number in a large hydraulic diameter annulus for the fully developed laminar flow with twisted tapes. The effect of the hydrostatic pressure on the friction factor is studied and found that the hydrostatic effects are significant for flow in large diameter annulus with low values of Reynolds numbers. Present study will be useful in design of heat exchangers with large hydraulic diameters.

INTRODUCTION

Laminar flow with heat transfer in straight annulus has wide applications in various fields of engineering, such as chemical industries, desalination plants, solar energy collectors, etc. To enhance the heat transfer rate in an existing heat exchanger, various methods have been reported in the literature [1-2]. The techniques of heat transfer augmentation with twisted tape and steel tape inserts have been used in many practical appliances [3-5].

Several papers on both the pressure drop and heat transfer characteristics using twisted tape insert is available [4-9]. Inserts in flow passage increase heat transfer rate at the cost of increase in pressure drop thereby demanding for more pumping power. Hence it is necessary to design the device with an optimization between the enhanced heat transfer rate and large pressure drop.

Witham [4] studied heat transfer enhancement by means of twisted-tape inserts way back at the end of the nineteenth century. Since then there have been several efforts in this direction. Royds[5], Date[6], Saha et al.[7], Saha and Dutta [8] studied the heat transfer and pressure drop in plane pipe flow with a small hydraulic diameter. Gupte and Date [5] presented the variation of friction factor and heat transfer rate in an annulus with small hydraulic diameter by employing twisted tape inserts. In all these studies, the variation of the friction factor was correlated to Reynolds number. Since, in a large hydraulic diameter pipe, hydrostatic pressure is one of the major parameters in addition to Reynolds number, these results deviate from reality for a large hydraulic diameter pipe. Hence there is a need for further study of pressure drop and heat transfer in large hydraulic diameter ducts.

In the present paper, an analysis has been carried out for large hydraulic diameter annulus correlating the friction factor with Reynolds number and hydrostatic pressure. An experimental set up has been constructed for the present study. Experiments were conducted with twisted tape inserts in the fully developed region of an annulus. The variation of the friction factor with Reynolds number along the circumferential direction has been discussed. Further, the effect of the hydrostatic pressure has been studied with different Reynolds number.

NOMENCLATURE

A	area of the crossessection perpendicular to the flow direction
a	dimensionless constant
b	dimensionless constant
c	dimensionless constant
D	diameter
f	friction factor
g	acceleration due to gravity
h	head
Ja	new dimensionless number (ratio of the dynamic head to shear stress due to hydrostatic effect)
l	length measured along the flow direction
Q	flow discharge
r	distance along the radial direction
Re	Reynolds number
V	average velocity of flow
Y	twist ratio (pitch of the twisted tape/hydraulic diameter)
	Greek Symbols
μ	coefficient of dynamic viscosity of fluid
ν	coefficient of kinematic viscosity of fluid
ρ	density of fluid
τ	shear stress direction
	Subscripts
h	hydraulic
hy	hydrostatic
i	inner
l	loss
o	outer
t	total
tt	twisted tape
z	axial direction
$ztot$	total

ANALYSIS

The Navier-Stokes equations for the steady and fully developed laminar flow in the cylindrical coordinates can be written as follows
Radial momentum:

$$0 = -\rho \times g \times sin\theta - \frac{\partial p}{\partial r} \tag{1}$$

Azimuthal momentum:

$$0 = -\rho \times g \times cos\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} \tag{2}$$

Streamwise momentum:

$$0 = -\frac{\partial p}{\partial z} + \frac{\mu}{r}[\frac{\partial (r\frac{\partial V_z}{\partial r})}{\partial r}] \tag{3}$$

where r, θ and z are the radial, azimuthal and streamwise directions respectively. p is pressure, ρ is fluid density, g is acceleration due to gravity and V_z is the velocity components along the axial direction. V_z varies along the radial direction only when the flow is fully developed inside a duct. Boundary conditions for an annulus are:

$$\begin{aligned} V_z &= 0 \ at \ r = r_i \\ V_z &= 0 \ at \ r = r_o \end{aligned} \tag{4}$$

where r_i is the outer radius of the inner pipe, and r_0 is the inner radius of the outer pipe. Velocity distribution V_z, in fully developed region of an annulus is obtained by solving Eq. (3) and using the boundary conditions (Eq. (4)). The final form is as follows:

$$V_Z = \frac{r^2 - r_i^2}{4\mu}\frac{\partial p}{\partial Z} + \frac{r_0^2 - r_i^2}{4\mu}\frac{\partial p}{\partial Z}\frac{\ln r + \ln r_i}{\ln \frac{r_0}{r_i}} \tag{5}$$

The velocity gradient in the developed region for an annulus can be obtained from Eq. (5) and is as follows:

$$\begin{aligned} \frac{\partial V_Z}{\partial r} &= \frac{r}{2\mu}\frac{\partial p}{\partial Z} + \frac{(r_0^2 - r_i^2)\frac{\partial p}{\partial z}}{4\mu r \ln \frac{r_0}{r_i}} \\ \Rightarrow \frac{\partial V_z}{\partial r} &= \frac{r}{4\mu}\frac{\partial p}{\partial z}(2 + \frac{r_0^2 - r_i^2}{r^2 \ln \frac{r_0}{r_i}}) \end{aligned} \tag{6}$$

In addition to the wall friction the pressure drop $\frac{\partial p}{\partial z}$ in an large diameter duct also depends on the velocity gradient generated by hydrostatic pressure ($\sqrt{2gr}$) variation along the radial direction. The total velocity distribution V_{ztot} in the fully developed region along the streamwise direction can be written as:

$$V_{ztot} = V_z + \sqrt{2gr} \tag{7}$$

where V_z and $\sqrt{2gr}$ are the velocity distributions due to the wall friction and hydro static pressure variation along the radial direction, respectively. For a small hydraullic diameter duct the effect of the term $\sqrt{2gr}$ can be neglected. However if the hydraullic diameter of the duct is large, the influence of the hydrostatic pressure on velocity distribution cannot be negelected. Hence the shear stress in large hydraullic diameter ducts is the sum of the shear stress produced by the velocity gradient due to hydrostatic pressure and the wall friction. The same is related to the total friction factor by the expression:

$$f = \frac{8\tau}{\rho V^2} = \frac{8\mu\frac{\partial V_{ztot}}{\partial r}}{\rho V^2} = \frac{8\mu(\frac{\partial V_z}{\partial r} + \sqrt{\frac{g}{2r}})}{\rho V^2} \tag{8}$$

In the Eq. (8) $V_z \sim V$, $r \sim D_h$ for velocity gradient due to the wall friction and $r \sim D_0$ for velocity gradient due to hydrostatic pressure variation along the radial direction. After these substitutions the expression for friction factor can be written as:

$$f = \frac{a\frac{\mu V}{D_h}}{\rho V^2} + \frac{b\mu\sqrt{\frac{g}{D_0}}}{\rho V^2} \tag{9}$$

In Eq. (9) a and b are the arbitrary constants. Now Eq. (9) can be written in terms of non-dimensional number as follows:

$$f = \frac{a}{Re} + \frac{b}{Ja} \tag{10}$$

where $Re = \frac{VD_h\rho}{\mu}$ is the Reynolds number and Ja is a newly defined nondimensional number indicating the ratio of the dynamic head to shear stress which can be represented mathematically as $Ja = \frac{\rho V^2}{\mu\sqrt{\frac{g}{D_0}}}$.

EXPERIMENTAL SETUP

Fig. 1 shows the experimental set-up for the present work. An overhead tank with 0.2 m^3 capacity serves as a constant head reservoir and is used to discharge the test liquid to the test section through a regulating valve. The test section consists of concentric straight pipes made of plexiglass, which are joined at regular 1m interval by flanges. The inner diameter of the outer pipe is 50mm and the annulus (flow passage) is of 30mm in the radial direction throughout. The small pipe is supported inside the large pipe centrally by means of two small concentric cylinders made up of plexiglass of 0.3mm thickness and 0.5mm length (Fig. 1). This concentric cylinder is supported circumferentially in the large pipe at each 2m distance by means of three thin pins of diameter 2mm. The test section is kept horizontal by aligning it using a spirit level.

Pressure taps were placed at a distance of 4.0, 4.25, 4.75m along the flow direction from the inlet valve position. At each location, three pressure taps were placed circumferentially at angles of 0^0, 120^0 and 240^0 measured from the top along the clockwise direction (Fig. 2). The pressure taps are 40mm long and made up of cast iron with inner diameter of 8mm and outer diameter of 10mm. The joints of the pressure taps and the pipe are sealed with an adhesive to ensure no leakage. The pressure drop reading from the pressure taps per unit length is found to be nearly same, i.e, $\frac{\Delta P_4 - \Delta P_4.25}{4.25-4} \approx \frac{\Delta P_4.25 - \Delta P_4.75}{4.75-4.25}$, hence it is concluded that the flow is fully developed.

All the pressure readings are taken under isothermal conditions. A U-tube manometer with water as

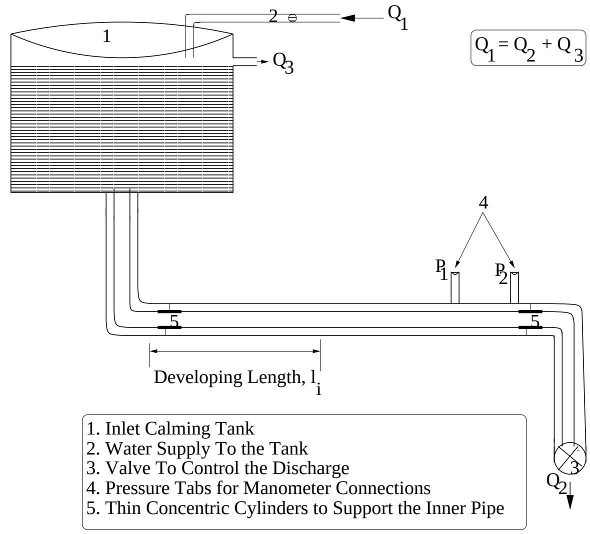

Figure 1: A schematic view of the experimental setup

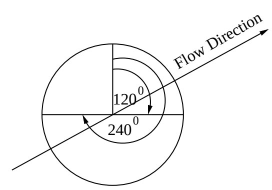

Figure 2: Angular positions for pressure taps and twisted tape inserts

working fluid is used for measuring the pressure drop at each probe location. The least count of the pressure measurement manometer is 0.5mm. Water is used as the working fluid for the entire study. Water is supplied to the pipe from a big reservoir of capacity 700 $litres$, which is maintained at a constant head. The discharge through the pipe is measured by means of a calibrated jar and stop watch.

Experimental data on pressure drop and discharge were collected under constant head conditions. At least two sets of data were collected to ensure the repeatability of the experiments. The average of the two sets of readings is used for analysis.

The developing length l_i for the laminar flow in an annulus is calculated using the following correlation [1]:

$$l_i = 0.056 D_h Re \tag{11}$$

where Reynolds number $Re = \frac{VD_h\rho}{\mu}$. The friction factor f in the developed region is calculated from the average pressure drop readings of the three circumferential pressure taps at three locations (4.0, 4.25 and 4.75m from the inlet valve location) as given

below,

$$f = \frac{2h_l g D_h A^2}{lQ^2} \quad (12)$$

where h_l is the head loss along the flow direction, g is the acceleration due to gravity, D_h is the hydraulic diameter. A is the area of the cross section perpendicular to the flow direction and Q is the flow discharge through the test section.

The ratio of dynamic pressure to the shear stress produced by the velocity gradient generated due to the hydrostatic pressure variation along the radial directon in the test section is calculated as

$$Ja = \frac{\rho Q^2}{\mu \sqrt{\frac{g}{D_O}}} \quad (13)$$

where D_o is the inner diameter of the outer pipe in the annulus. In case of plain pipe, it represents the hydraulic diameter D_h.

The Reynolds number is calculated based on discharge Q through the test section

$$Re = \frac{QD_h}{A\nu} \quad (14)$$

where ν is the kinematic viscosity of the working fluid.

Twisted tapes of two different twist ratios ($Y =$ 8.67 and 9.23) were fabricated from thin sheet metal ($1mm$ thickness) with smooth surfaces by attaching one end of the sheet metal strip of width $27mm$ (nearly equal to the hydraulic diameter of annulus) to the sealing and twisting the other end with a heavy load attachment to prevent the bending of the metal strip.

RESULTS AND DISCUSSION

In the present work a series of experiments were carried out to study the friction factor variation in a large hydraulic diameter annulus with various insert parameters and different number of twisted tapes in an annulus for the fully developed laminar flow. The laminar flow (Re varies from 40 to 2000) has been chosen in the present work because the hydrostatic effects are important when the Reynolds number is small. Twisted tapes of two different twist raios $Y = 8.67$ and 9.23 and two different lengths l_{tt}=62cm and 30cm have been selected for the study.

The variation of non-dimensional parameter Ja with Reynolds number Re for various outer diameters of annulus with same $\frac{d_i}{d_o} = 0.4$ is plotted in Fig.3. From Fig. 3 it is clear that the value of Ja is large for small values of d_o, for constantRe and$\frac{d_i}{d_o}$. Further, it is observed from Eq.(10)that the term Ja increases

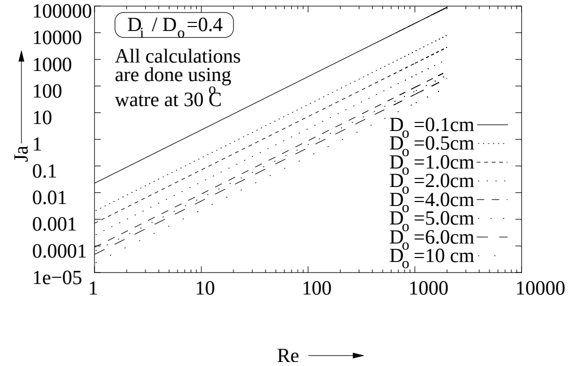

Figure 3: Variation of Ja with Reynolds number Re

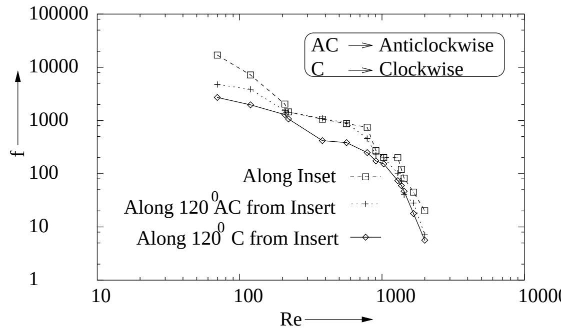

Figure 4: Variation of friction factor f with Reynolds number Re in an annulus with single twisted tape ($Y = 8.67$ and $l_{tt} = 62\ cm$) insert along 120^0 in streamwise direction

with decrease in friction factor f. Therefore, it is obvious that for small hydraulic diameter pipes and annulus, the friction factor will not be affected much by the parameter Ja. However, in case of large hydraulic diameter pipes the phenomenon is reversed and the influence of the nondimensional number Ja on friction factor f dominates compared to the Re. Hence the results reported in literature for friction factors and correlated in terms of Reynolds number cannot be used directly for large hydraulic diameter ducts.

Fig. 4 shows the variation of the friction factor f with Re for a single twisted tape insert ($Y = 8.67$ and $l_{tt} = 62\ cm$) at 120^0 along the test section. From Fig. 4 it is clear that the friction factor is varying with circumferential direction θ. Hence in an annulus with twisted tape insert, the pressure drop will vary along the circumference. It is also clearly realized from the figure that the pressure drop is large along the direction of the twisted tape. Further, it is found that the friction factor is not a constant function of Reynolds number along the circumference of the duct

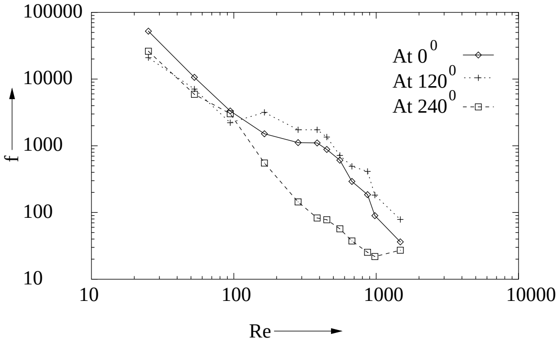

Figure 5: Variation of friction factor f with Reynolds number Re in an annulus with single twisted tape ($Y = 9.23$ and $l_{tt} = 30\ cm$) insert along 120^0 in streamwise direction

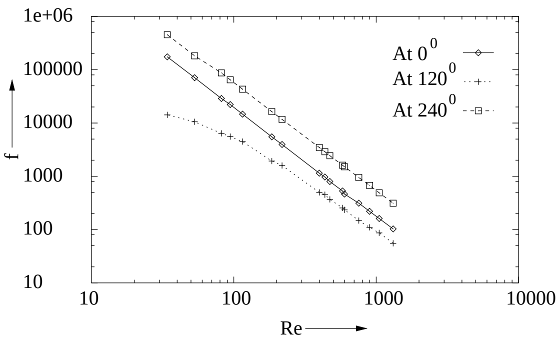

Figure 6: Variation of friction factor f with Reynolds number Re in an annulus with two twisted tapes ($Y = 9.23$ and $l_{tt} = 30\ cm$) insert along 120^0 and 240^0 in streamwise direction

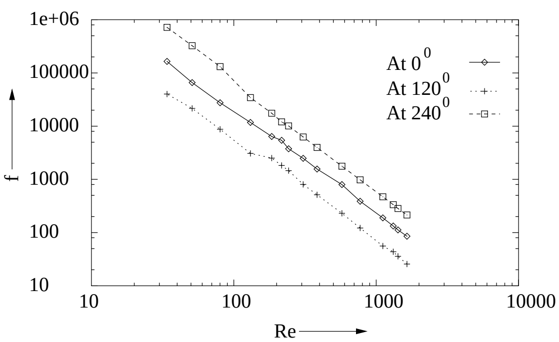

Figure 7: Friction factor f variation with Reynolds number Re in an annulus with three twisted tapes ($Y = 9.23$ and $l_{tt} = 30\ cm$) insert along 0^0, 120^0 and 240^0 respectively.

θ.

The variation of friction factor with Re for a single twisted tape insert ($Y = 9.23$ and $l_{tt} = 30\ cm$) at

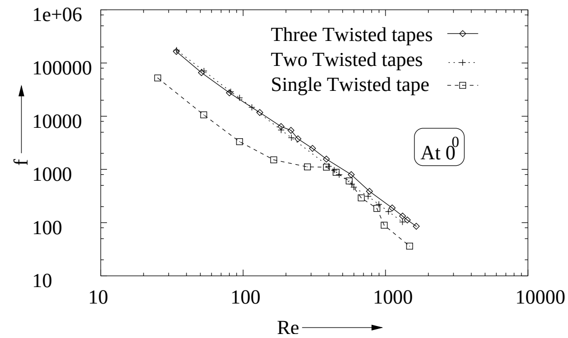

Figure 8: Comparison of friction factor f with Reynolds number Re for different number of twisted tapes at 0^0.

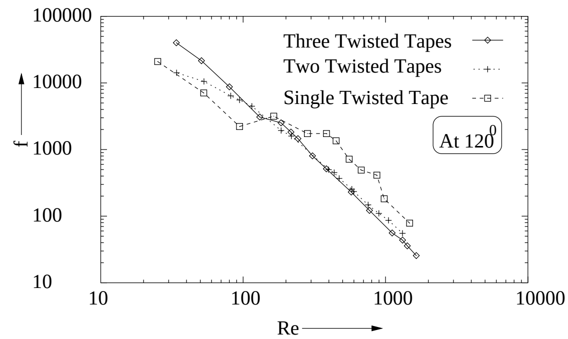

Figure 9: Comparison of friction factor f with Reynolds number Re along 120^0 for different twisted tapes.

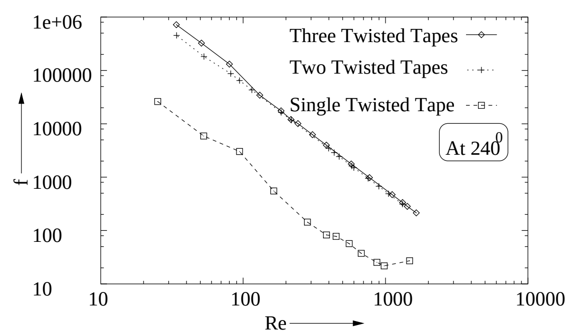

Figure 10: Comparison of friction factor f with Reynolds number Re along 240^0 for different number of twisted tapes.

120^0 along the test section is plotted in Fig. 5. It can be seen from Fig. 5 that the pressure drop is large along 0^0 in low Reynolds number range ($Re < 100$) and as Reynolds number increases, the pressure drop along the 120^0 exceeds that along 0^0. Moreover, the pressure drop along 240^0 is found to be small

throughout the Re range.

The variation of friction factor with Reynolds number for two twisted tapes insert ($Y = 9.23$ and $l_{tt} = 30\ cm$) at 120^0 and 240^0 along the testsection is shown in Fig. 6. From this figure it is observed that the friction factor varies almost linearly with Reynolds number, throughout the tested range except for small Reynolds number along 0^0. It is also observed that the values of the friction factor are quite large along the twisted tape inserts (120^0 and 240^0) as compared to 0^0.

Figure. 7 shows the variation of friction factor with Reynolds number for three twisted tapes inserts ($Y = 9.23$ and $l_{tt} = 30\ cm$) at 120^0, 240^0 and 0^0 along the testsection. Here the friction factor is found to vary almost linearly along the three angles considered. However the pressure drop is found to be highest at 240^0 along the test section, followed by 0^0 and 120^0.

The comparison for friction factor variation along 0^0 for single, two, and three twisted tapes inserts ($Y = 9.23$ and $l_{tt} = 30\ cm$) is shown in Fig. 8. It is found that the friction factors for the three twisted tape inserts is quite high as compared to the single twisted tape insert throughout the tested Reynolds number range. However the difference in the friction factor for two twisted tape and three twisted tape insert is found to be insignificant.

Figure. 9 represents the variation of friction factor at 120^0 along the test section for single, two and three twisted tapes inserts ($Y = 9.23$ and $l_{tt} = 30\ cm$) along the testsection. Here it is observed that for $Re < 300$, the values of friction factor are quite high for the three twisted tapes inserts followed by the single twisted tape and two twisted tape inserts respectively.

A comparison of friction factor variation along 240^0 for single, two and three twisted tapes ($Y = 9.23$ and $l_{tt} = 30\ cm$) insert is plotted in Fig. 10. From this figure it can be easily pointed out that the friction factor for three and two twisted tape inserts are almost same, whereas the friction factor for the single twisted tape insert are found to be much smaller in comparison to the other two cases.

CONCLUSIONS

For large hydraulic diameter annulus with twisted tape inserts the following points may be concluded:

1. The friction factor depends on hydrostatic pressure, as well as with the Reynolds number.
2. The friction factor varies circumferentially.
3. The friction factor is not a constant function of Reynolds number.
4. The friction factor increases with increase in number of inserts, however the increment in friction factor for two twisted tapes to three twisted tapes insert is not significant.
5. The friction factor is found to be large along the twisted tape insert.
6. The present study will be useful to do further research in large diameter ducts.

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